Non-contact interactions and the hadronic light-by-light contribution to the muon $g-2$ *

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We summarise recent results for the quark loop part of the light-by-light scattering contribution to the muons anomalous magnetic moment. In particular we focus on the impact of a momentum dependent quark and quark-photon vertex. We compare the Dyson-Schwinger description with that of the extended Nambu–Jona-Lasinio model (ENJL) and find important quantitative differences. In particular the transverse parts of the quark-photon-vertex, which serve as a dynamical extension of simple vector meson dominance models, do not yield the large suppression as found in the ENJL model.

1. Introduction

Here we give a brief summary of our results on the anomalous magnetic moment of muon [1, 2, 3]. This quantity serves to provide a precision test of the standard model, in particular the electromagnetic [4], weak [5] and strong force. The QED contributions are dominant followed by QCD, with the latter dominating the theoretical error.

These QCD corrections are the leading order hadronic vacuum polarisation (LOHVP) [6] and the hadronic light-by-light scattering contribution (HLBL). The former may be inferred from experiment [7], with Lattice results also becoming competitive [8, 9, 10]. The HLBL contribution can only be determined from theory, with many models attempting its evaluation [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 1]. We focus here on two approaches: the Extended Nambu–Jona-Lasinio (ENJL) model [11]; and the Dyson-Schwinger equations (DSEs) [19, 20, 1].

The combined theory result stands at $116\,591\,827.0\,(64)\times 10^{-11}$[21], which compares to the experimental result of $116\,592\,089\,(63)\times 10^{-11}$ [22, 23]. The discrepancy stands at 3σ, but may be as high as $\sim 4.8\sigma$ [24].

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One interpretation of this discrepancy may be a signal of beyond the standard model physics. In these proceedings we argue a note of caution should be taken with the present HLBL estimation and its error. We believe that the limitations of models thus far used lead to an overly optimistic value. To demonstrate, we compare our approach with the ENJL and highlight the differences and consequences in the quark-loop contribution,. In particular, we show that the large suppression due to VMD in the ENJL [11] approach is an artefact of the contact interaction therein.

We will employ the Dyson-Schwinger equations (DSEs), which are renormalisable functional integral relations amongst the Green's functions of the QFT. To satisfy the vector and axial-vector Ward-Takahashi identities (WTI) we take the rainbow-ladder truncation [25, 26]. This is successful in a wide range of meson [25, 26, 27, 28, 29] and baryon [30, 31, 32, 33] properties. For a summary of the LOHVP contribution calculated in the DSE approach, compared with recent Lattice QCD results, see Ref. [3].

2. Comparison: DSE vs. ENJL

First, we note that the DSEs are renormalisable and feature a continuous connection between the infrared and ultraviolet limit. This contrasts with ENJL which is a non-renormalisable effective model with a cut-off on the order of 1 GeV. Secondly, due to its contact interaction the ENJL model features dressing functions with trivial momentum dependence. The DSE approach is quite different in this respect; see Ref. [1] for more details.

The inverse quark propagator is

$$S^{-1}(p) = Z_f^{-1}(p^2)\left(-i\not{p} + M(p^2)\right), \tag{1}$$

with mass function $M(p^2)$ and wave-function renormalisation $Z_f(p^2)$. In the DSE approach both feature a momentum dependence that connects the perturbative ultraviolet limit with the non-perturbative deep infrared. The ENJL approach [11, 34] is a low energy effective theory with a contact interaction and so we have $Z_f = 1$ and a constituent quark mass $M(p^2) \approx$ 300 MeV. In the DSE approach $Z_f(p^2) < 1$ leading to a suppression of the propagator that is compensated by a comparable enhancement in the quark-photon vertex as constrained by the Ward-Takahashi identity (WTI)

$$\imath P_\mu \Gamma_\mu(P,k) = S^{-1}(k_-) - S^{-1}(k_+) \ . \tag{2}$$

That the DSE mass function connects the current quark mass (a few MeV) to the constituent-like mass (a few hundred MeV) suggests that on average a lighter than constituent quark mass is probed, typically of order 200 MeV. This is the main reason for the larger relevance of this diagram in our approach.

The quark-photon vertex is more complicated. Both approaches satisfy the WTI and in addition feature dynamical vector meson poles. However, in the ENJL approach the contact interaction decouples loop integral corrections and the bubble sum may be determined from a geometric series. The vertex has the form

$$\Gamma_\mu^{\mathrm{ENJL}} = \gamma_\mu - \gamma_\mu^T \frac{Q^2}{Q^2 + M_V^2} , \tag{3}$$

which contains the bare vertex γ_μ and the leading transverse structure $\gamma_\mu^{\mathrm{T}} = (\delta_{\mu\nu} - Q_\mu Q_\nu/Q^2)\gamma_\nu$. The dressing of this transverse part is given here in the VMD limit of the ENJL model [34] where for two flavours M_V is identified with the ρ-mass. Using the transversality of the hadronic photon four-point function with respect to its photon legs, this vertex can be reduced to $\gamma_\mu M_V^2/(Q^2 + M_V^2)$ which highlights the strong suppression induced by the VMD contribution to the vertex.

In the DSE approach, the interaction type is non-contact and the vertex is decomposed into twelve tensor components

$$\Gamma_\mu(Q,k) = \sum_{i=1}^{4} \lambda^{(i)}(Q,k) L_\mu^{(i)} + \sum_{i=1}^{8} \tau^{(i)}(Q,k) T_\mu^{(i)} , \tag{4}$$

where k, Q are the relative and quark momentum and the total photon momentum, respectively. The vector meson bound state appears dynamically in the transverse vertex structure. A simple fit to the numerical results of the quark-photon vertex has been provided [27]

$$\Gamma_\mu(Q,k) \simeq \Gamma_\mu^{\mathrm{BC}} - \gamma_\mu^{\mathrm{T}} \frac{\omega^4 N_V}{\omega^4 + k^4} \frac{f_V}{M_V} \frac{Q^2}{Q^2 + M_V^2}\, e^{-\alpha(Q^2 + M_V^2)}, \tag{5}$$

which consists of the Ball-Chiu part, Γ_μ^{BC}, and the leading transverse structure corresponding to $T_\mu^1 = \gamma_\mu^{\mathrm{T}}$. Good agreement with the numerical solution is found for the parameters$\omega = 0.66\,\mathrm{GeV}$, $\alpha = 0.15$ and $N_V f_V/M_V = 0.152$. Note that, as in the ENJL model, we have in Eq. (5) a part that is given via the WTI (the BC vertex) and a transverse part.

We immediately see a large difference between the transverse parts of the two quark-photon vertices. In the ENJL approach there is only a dependence on Q, the total momentum of the photon, whereas in the DSE approach the relative quark momentum k is also a parameter. By comparison, one may *restore* this dependence on the relative quark momentum by introducing the function

$$f(k^2) = \frac{\omega^4}{k^4 + \omega^4} . \tag{6}$$

Similarly, one could simulate the impact of neglecting the relative quark momentum in the DSE approach by setting $k = 0$ explicitly. These two cases are shown in Table 1.

Without relative momentum		With relative momentum	
$\gamma^T_{\mu\,\mathrm{ENJL}}$	43	$\gamma^T_{\mu\,\mathrm{ENJL}} f(k^2)$	103
$\gamma^T_{\mu\,\mathrm{DSE,fit}}(k=0)$	43	$\gamma^T_{\mu\,\mathrm{DSE,fit}}$	105
$\gamma^T_{\mu\,\mathrm{DSE,calc}}(k=0)$	41	$\gamma^T_{\mu\,\mathrm{DSE,calc}}$	96

Table 1. Impact of restricting/restoring the relative quark momentum in the leading transverse part of the quark-photon vertex for the ENJL model, the DSE fit of Eq. (5) and the calculated quark-photon vertex (DSE,calc), on the quark-loop contribution to a_μ. Units are $\times 10^{-11}$, and we use two light-quark flavours.

The results clearly show that restricting the relative quark momentum to be identically zero leads to a significant suppression of the contribution, from ~ 100 MeV down to ~ 40 MeV.

Thus, the combination of both a dynamical quark mass and a quark-photon vertex with realistic momentum dependence yields enhancement of the quark-loop contribution to hadronic light-by-light scattering in the muon $g-2$ as compared with other effective models such as ENJL.

3. Conclusions

We presented a summary of our comparison between the DSE approach and that of the ENJL model with regards to the quark-loop contribution to hadronic light-by-light scattering in the muon $g-2$ [1]. There are important differences due to momentum dependence.

For the quark mass function, the connection between light current quark masses at large momenta and heavy constituent quark-masses at small momenta entails from the mean-value theorem that some average mass in-between is probed. This average mass is of the order 200 MeV, far lighter than typical values considered in say the ENJL model. Note that this does not imply a constituent quark mass of ~ 200 MeV since it is a merely an integrand-weighted average.

In the quark-photon vertex, we found that restricting the relative quark-momentum dependence to be zero, a natural consequence in the ENJL model due to the contact interaction, yields important quantitative differences as to the impact of dynamical vector meson poles. The suppression of the quark-loop reported in the ENJL model is an artefact of this momentum restriction, and is almost completed mitigated within the DSE approach due to the full momentum dependence of the vertex considered therein.

Thus, we conclude that the standard value $a_\mu^{LBL} = 105(26) \times [10^{-11}]$ used in current evaluations of the anomalous magnetic moment of the muon [6, 24] may be too small concerning its central value and is probably much too optimistic in its error estimate.

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